EE 435
Lecture 43

Phased Locked Loops and VCOs

Final Exam:

Scheduled on Final Exam Schedule:

Wednesday May 14 7:30 a.m.

Revised Final Exam:

- Take-home format – open book and open notes
- Will be posted on course WEB site by Wednesday May 7
- Due at 9:30 a.m. on Wednesday May 14 : Provide hard copy and a backup solution as a pdf file in Canvas
- Honor system – students must agree to not discuss the exam with anyone except the course instructor or TA, not post or distribute the exam to any third party, and not use any outside resources (such as Chegg) for solving any problems on the exam.

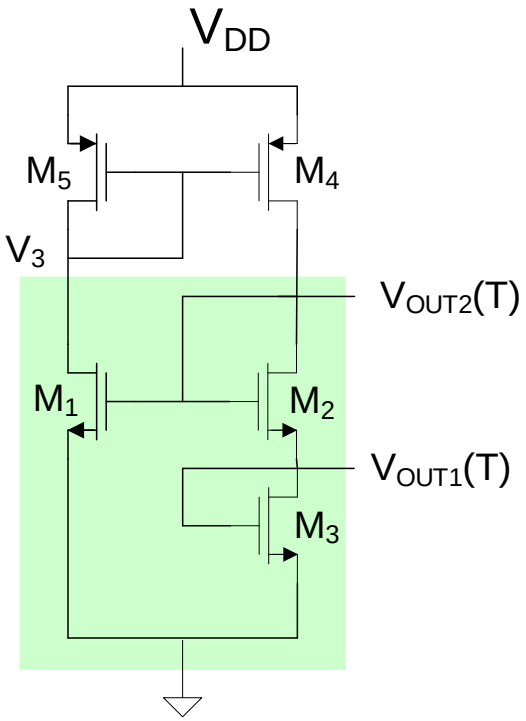
If anyone has any constraints that makes it difficult to work with this revised format or would like to do an in-person exam from 7:30 to 9:30 on May 14, please contact Professor Geiger by 5:00 p.m. on Wednesday May 7

Review from last lecture

Bias Voltages/Currents Generators

How are these voltages and currents generated?

Voltage Outputs:



Inverse-Widlar

$$V_{01} = V_{Tn}\left(\frac{1-\sqrt{\frac{M_{54}W_2L_1}{W_1L_2}}}{1+\sqrt{\frac{W_2L_3}{W_3L_2}}-\sqrt{\frac{M_{54}W_2L_1}{W_1L_2}}}\right)$$

$$V_{02} = V_{Tn}\left(\frac{1+\sqrt{\frac{W_2L_3}{W_3L_2}}-2\sqrt{\frac{M_{54}W_2L_1}{W_1L_2}}}{1+\sqrt{\frac{W_2L_3}{W_3L_2}}-\sqrt{\frac{M_{54}W_2L_1}{W_1L_2}}}\right)$$

M_{54} is the M_5:M_4 Current Mirror Gain

Supply-independent Bias Generator!

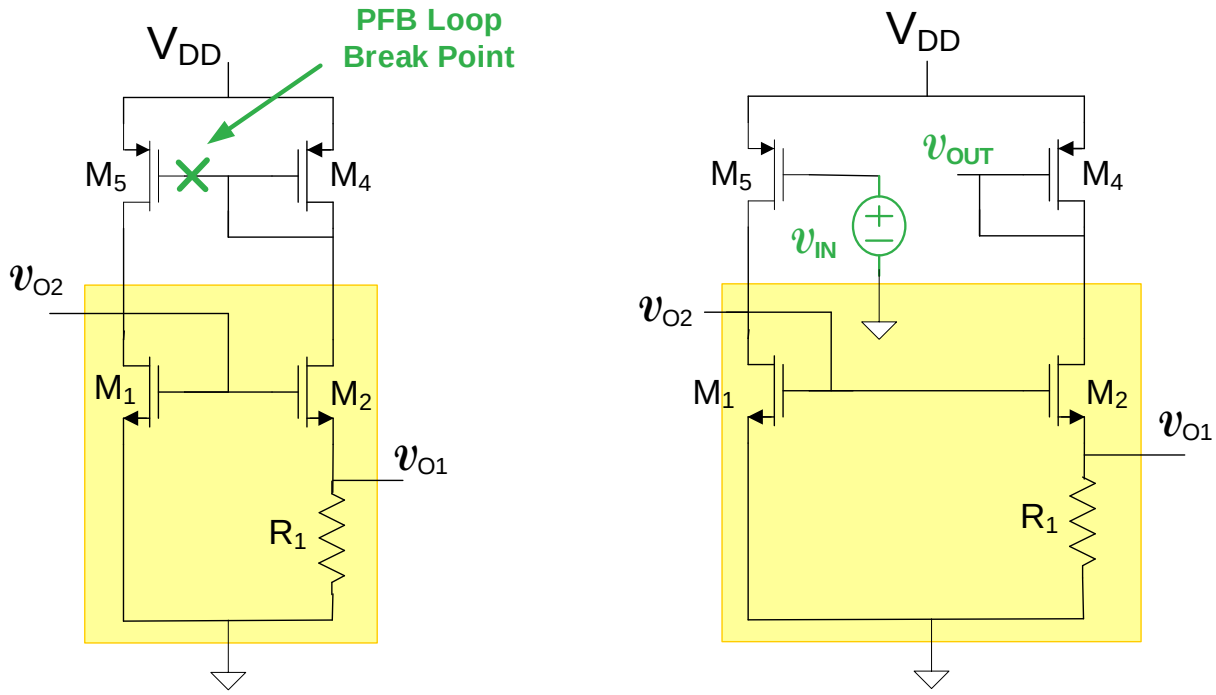
Start-up circuit needed (notice positive feedback loop)

Supply-independent Bias Generators Widely Used

Review from last lecture

Bias Voltages/Currents Generators

Need for Start-up Circuit

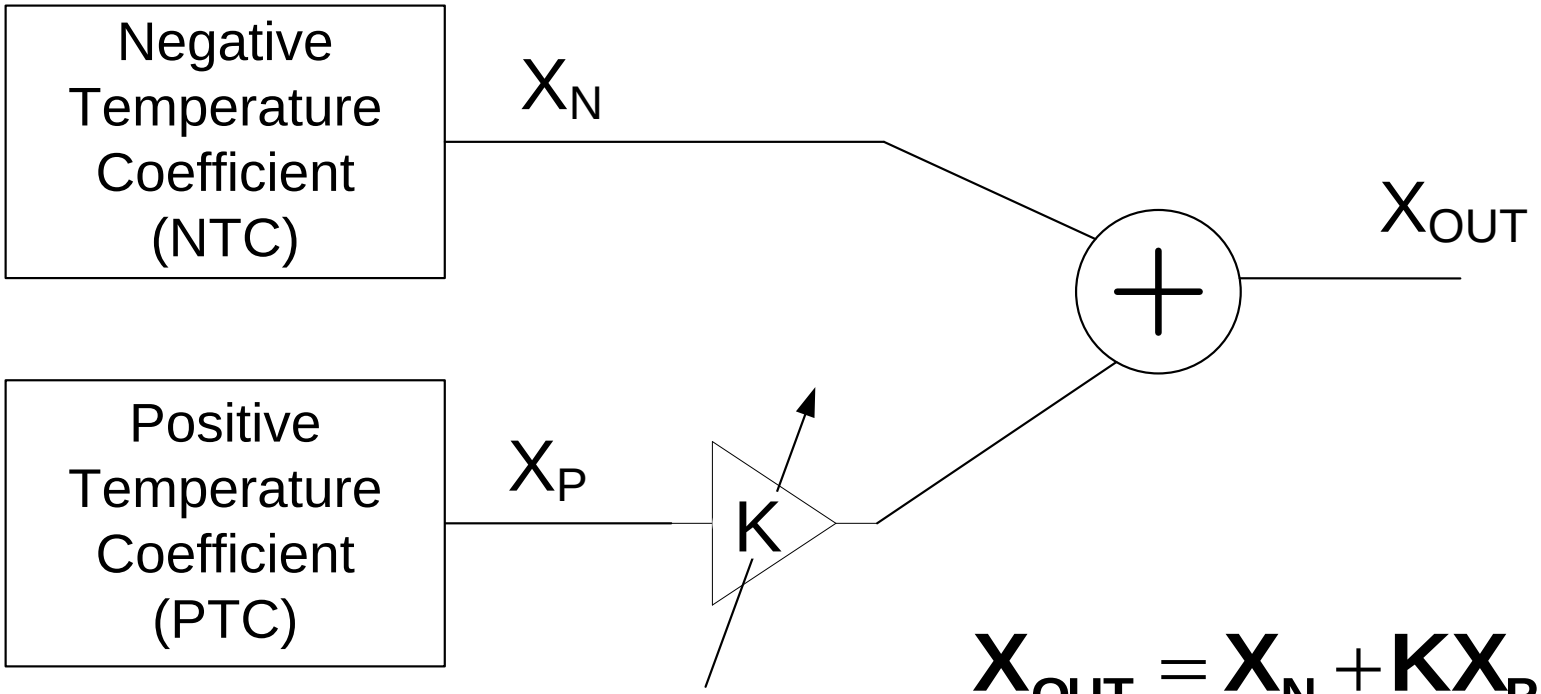


$V_{OUT}=f(V_{IN})$ termed the return map

Termed Homotopy Analysis

Must not perturb operating point when breaking loop !

Standard Approach to Building Voltage References

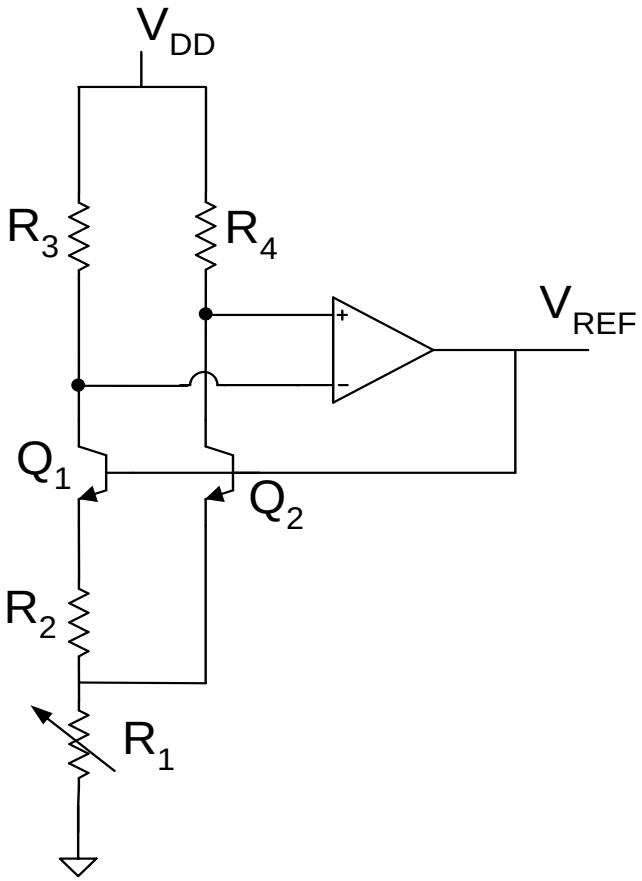


$$\mathbf{X_{OUT} = X_N + KX_P}$$

Pick K so that at some temperature T_0, $\left.\frac{\partial(\mathbf{X_N + KX_P})}{\partial \mathbf{T}}\right|_{\mathbf{T}=\mathbf{T_0}} = \mathbf{0}$

Review from last lecture

Early Bandgap Reference (and still widely used!)

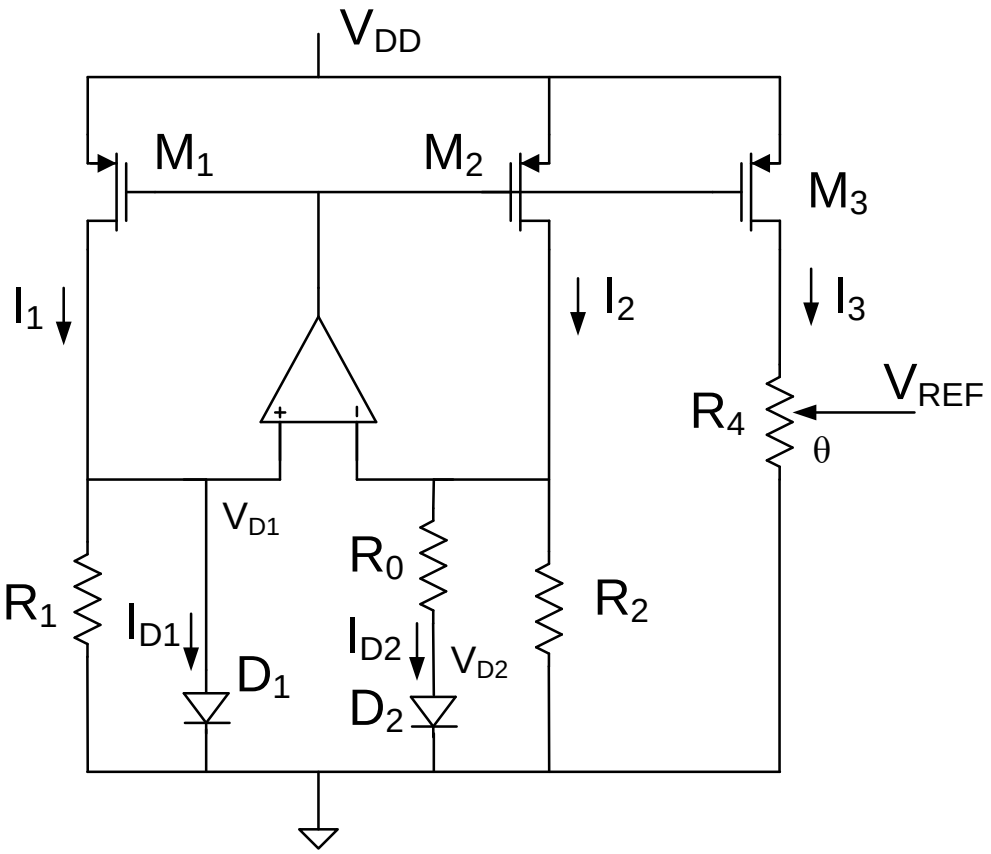


P.Brokaw, "A Simple Three-Terminal IC Bandgap Reference", IEEE Journal of Solid State Circuits, Vol. 9, pp. 388-393, Dec. 1974.

- Brokaw coined term "bandgap reference" when referring to this circuit
- Properties very similar circuits introduced by Widlar and Kujik a small while earlier
- Paper submitted May 1974, Widlar paper submitted March 1970

Review from last lecture

Banba Bandgap Reference



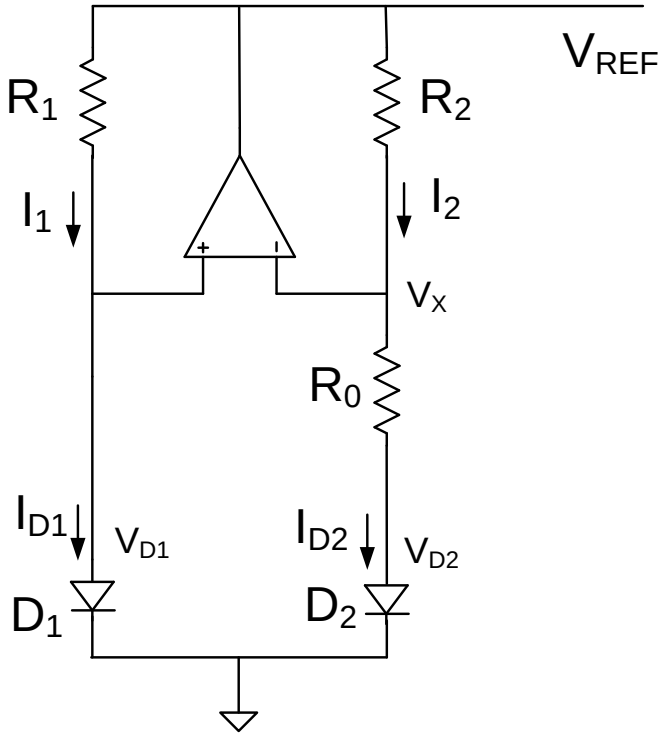
$$\frac{I_{D2}}{I_{D1}} = \text{constant}$$

[7] H. Banba, H. Shiga, A. Umezawa, T. Miyaba, T. Tanzawa, A. Atsumi, and K. Sakkui, IEEE Journal of Solid-State Circuits, Vol. 34, pp. 670-674, May 1999.

Note this was introduced 25 years after the Brokaw reference

Review from last lecture

Kujik Bandgap Reference



$$I_{R0} = \frac{\Delta V_{BE}}{R_0}$$

$$I_2 = I_{R0}$$

$$V_{REF} = I_2 R_2 + V_{BE1}$$

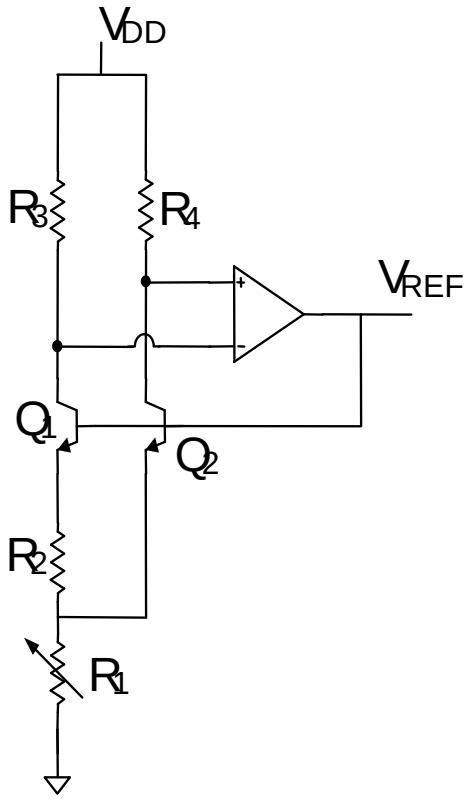
solving, we obtain

$$V_{REF} = \frac{R_2}{R_0}\Delta V_{BE} + V_{BE1}$$

$$V_{REF} = a_{22} + b_{22}T + c_{22}T\ln T$$

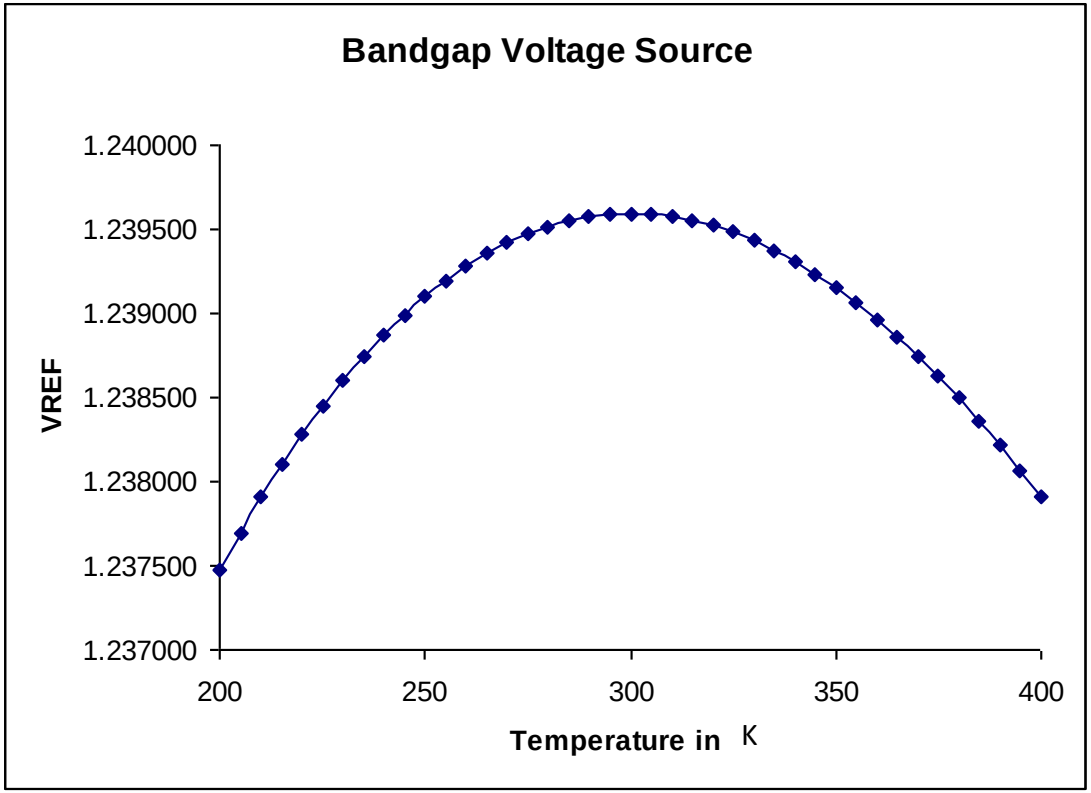
Review from last lecture

First Bandgap Reference (and still widely used!)



$$V_{REF} = a_1 + b_1 T + c_1 T \ln T$$

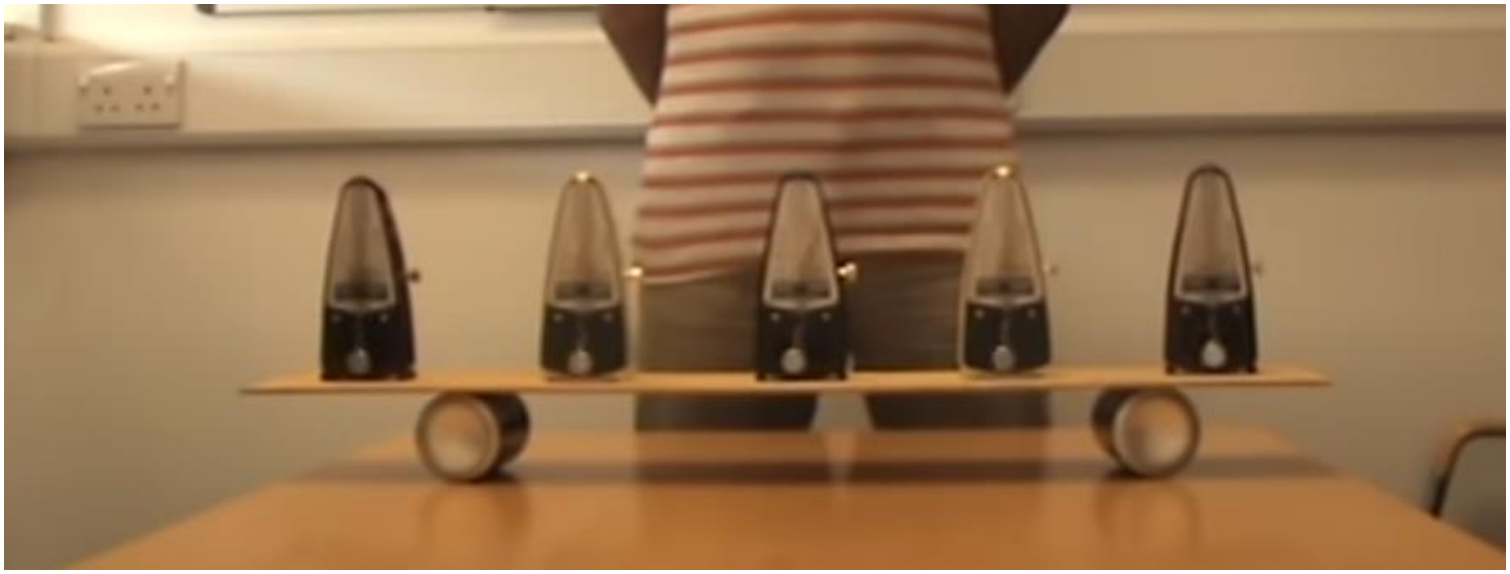
$$V_{REF}(T_{INF}) = V_{G0} + \frac{kT_{INF}}{q}(m-1)$$



VGO	1.206
TO	300
VBEO2	0.65
m-1	1.3
k/q	8.61E-05

Only 2mV change over 200°C temp range !

Injection Locking



https://www.youtube.com/watch?v=W1TMZASCR-I

Phase Locked Loops

Special case of injection-locked systems

In 1600's pendulums and organ pipes were occasionally injection locked

In 1919 electronic oscillators were observed that lock or sychronize

For the subsequent several decades, a form of PLLs was used to build radio and television receivers

In 1969 Signetics introduced the first integrated PLLs (NE 565 and NE 567)

In 1970's RCA introduced the 4046 PLL

ON Semiconductor HC4046A

Phase Locked Loops

PLLs widely used throughout industry today

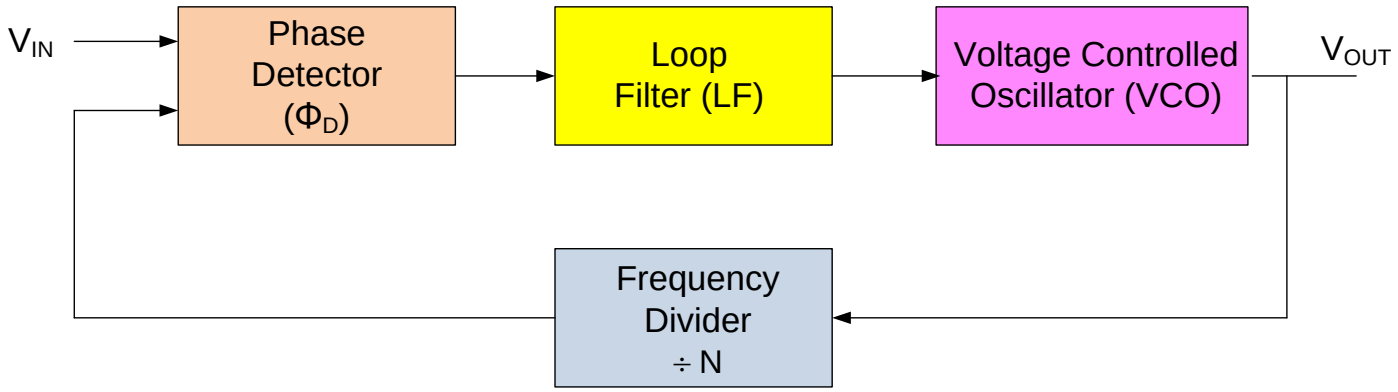
Easy to establish basic operation of a PLL

Highly nonlinear systems with seemingly simple operation but high-end designs can become quite complicated and mathematical rigor is often cumbersome and only approximate

Performance potential of most wireless communication systems and serial data transmission networks strongly dependent upon the performance characteristics of multiple embedded PLLs

Concepts of DLL and PLL are closely related

Basic PLL Architecture

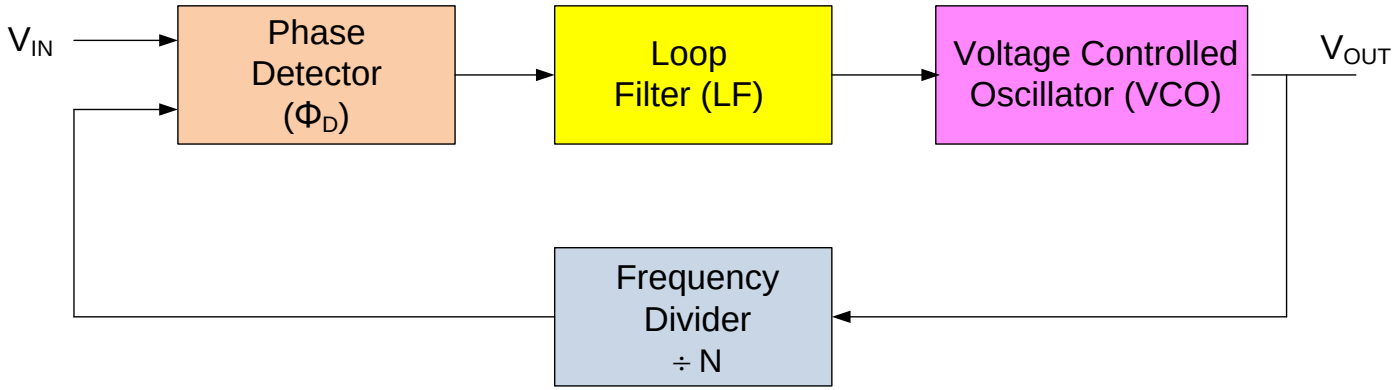


Applications include:

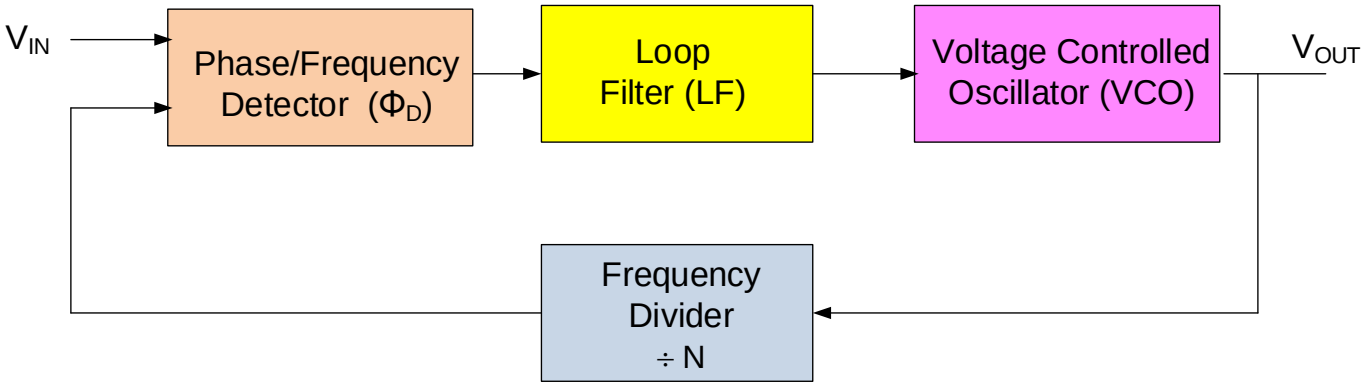
- Frequency Demodulation
- Frequency Synthesis
- Clock Synchronization
- Noise filtering (extreme)
- Tracking and calibrated filters
- …..

- **One of the most widely used analog blocks**
- **Many SoC systems include multiple PLLs**
- **Closely related to Delay Locked Loop (DLL)**

Basic PLL Architecture

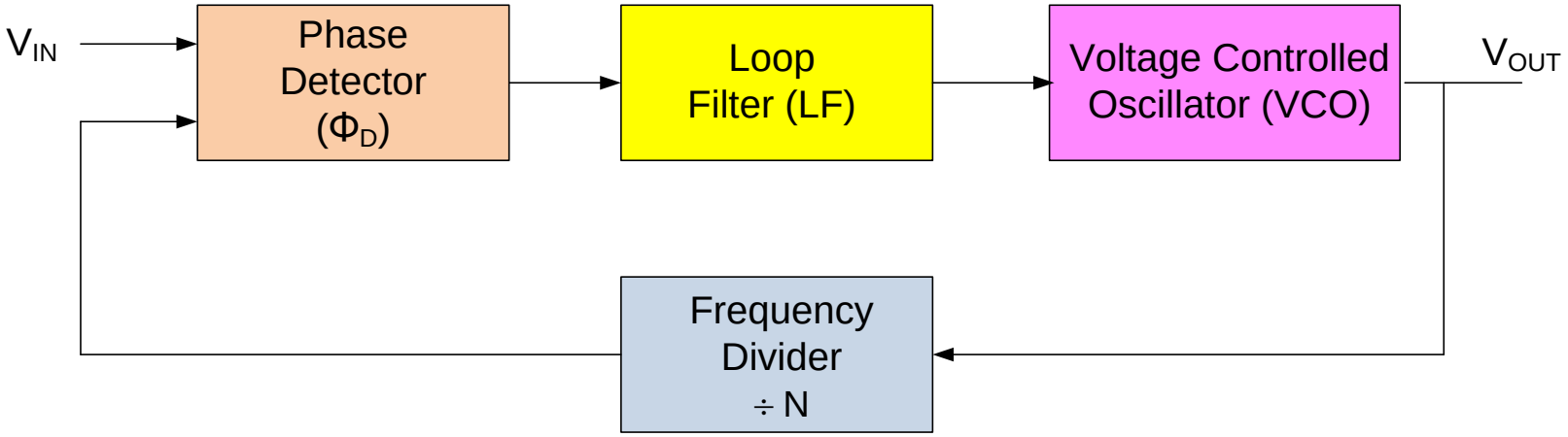


Most basic structure actually uses phase/frequency detector but still termed PLL[1]



[1]Ian Collins, "Phase-Locked Loop (PLL) Fundamentals", Analog Dialogue, July 2018.

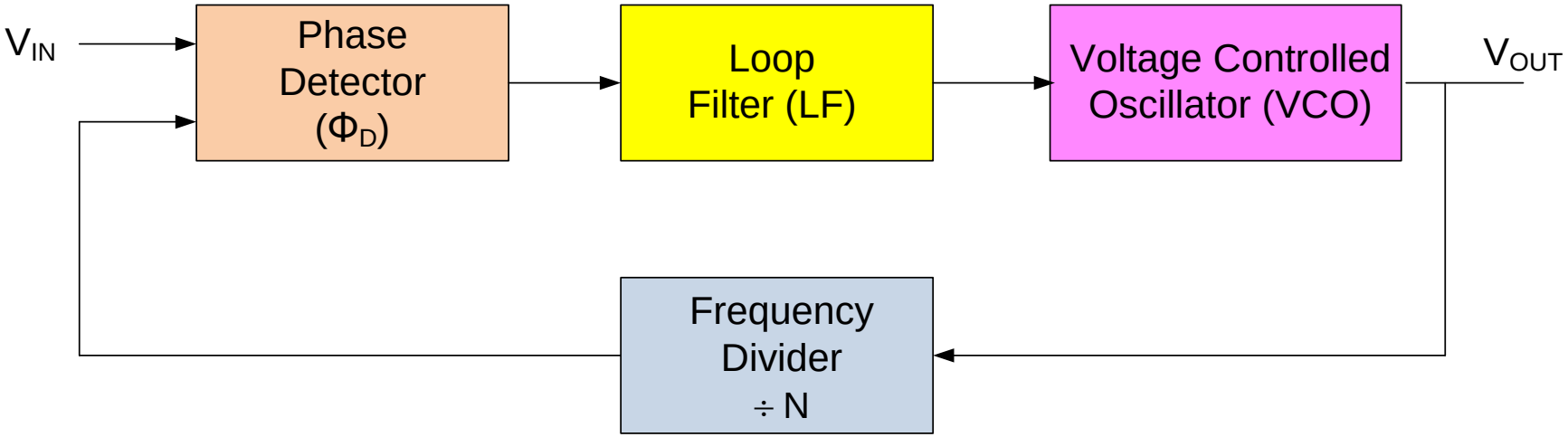
Basis PLL Architecture



Applications by subcategory:

- Clock and Data Recovery
- Recovering signals when SNR <<<1
- Timing generators in digital systems

Desired Operation of PLL



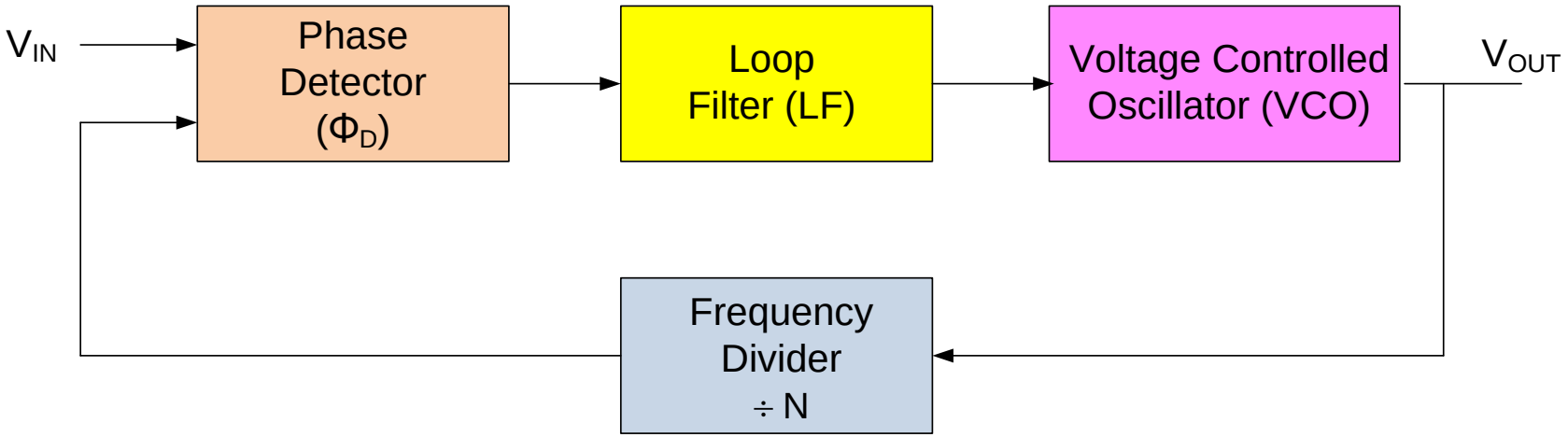
$$V_{IN}=V_M\sin(\omega_{IN}t+\phi_{IN})$$

Desired output when locked:

$$V_{OUT}=V_X\sin(N\omega_{IN}t+\phi_{OUT})$$

- Relationship between V_M and V_X is of little concern
- Frequency relationship is critical
- ϕ_{OUT} is often critical too
- Waveshape of V_{IN} and V_{OUT} is often of little concern
  May be highly distorted or even square waves

Desired Operation of PLL



$V_{IN}=V_M\sin(\omega_{IN}t+\phi_{IN})$

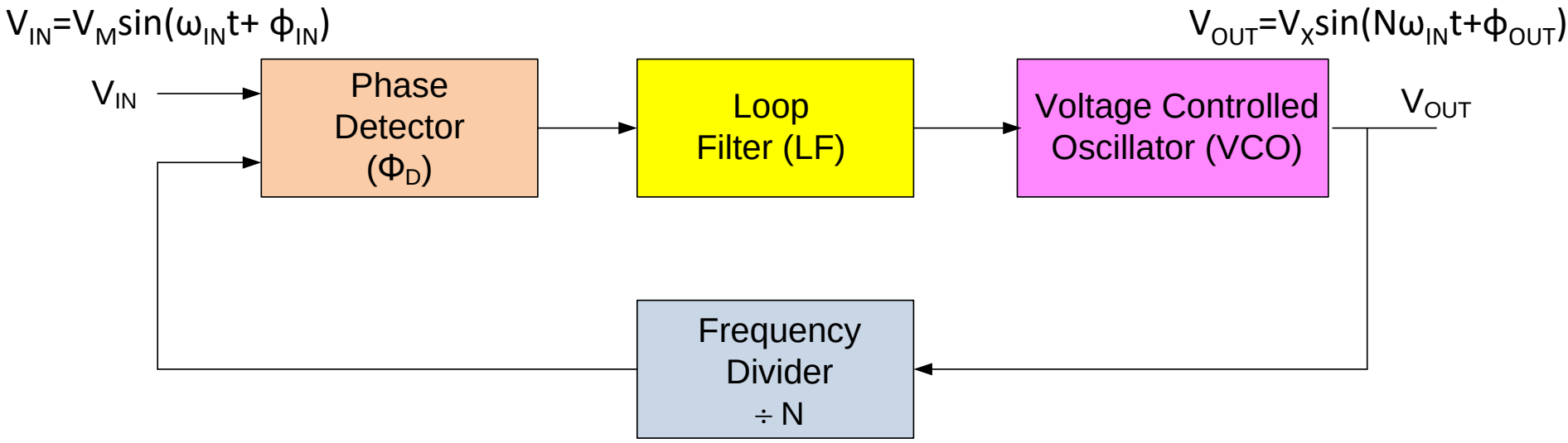
Desired output when locked:

$V_{OUT}=V_X\sin(N\omega_{IN}t+\phi_{OUT})$

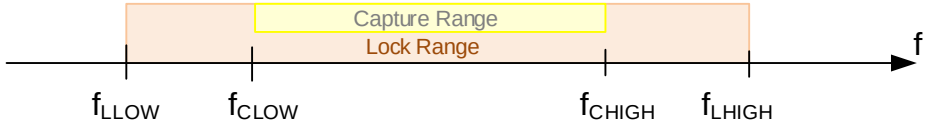
Some Terminology of PLLs

- Locked / Unlocked — Locked when V_{OUT} assumes desired value
- Lock Range — $f_{LLOW} < f_{IN} < f_{LHIGH}$ If locked, will remain locked for f_{IN} in lock range
- Capture Range — $f_{CLOW} < f_{IN} < f_{CHIGH}$ If unlocked, will lock for f_{IN} in capture range
- Free-running frequency — frequency of VCO when not locked
- Harmonic/Subharmonic Lock

Desired Operation of PLL



Capture range always less than lock range $f_{LLOW} < f_{CLOW} < f_{CHIGH} < f_{LHIGH}$

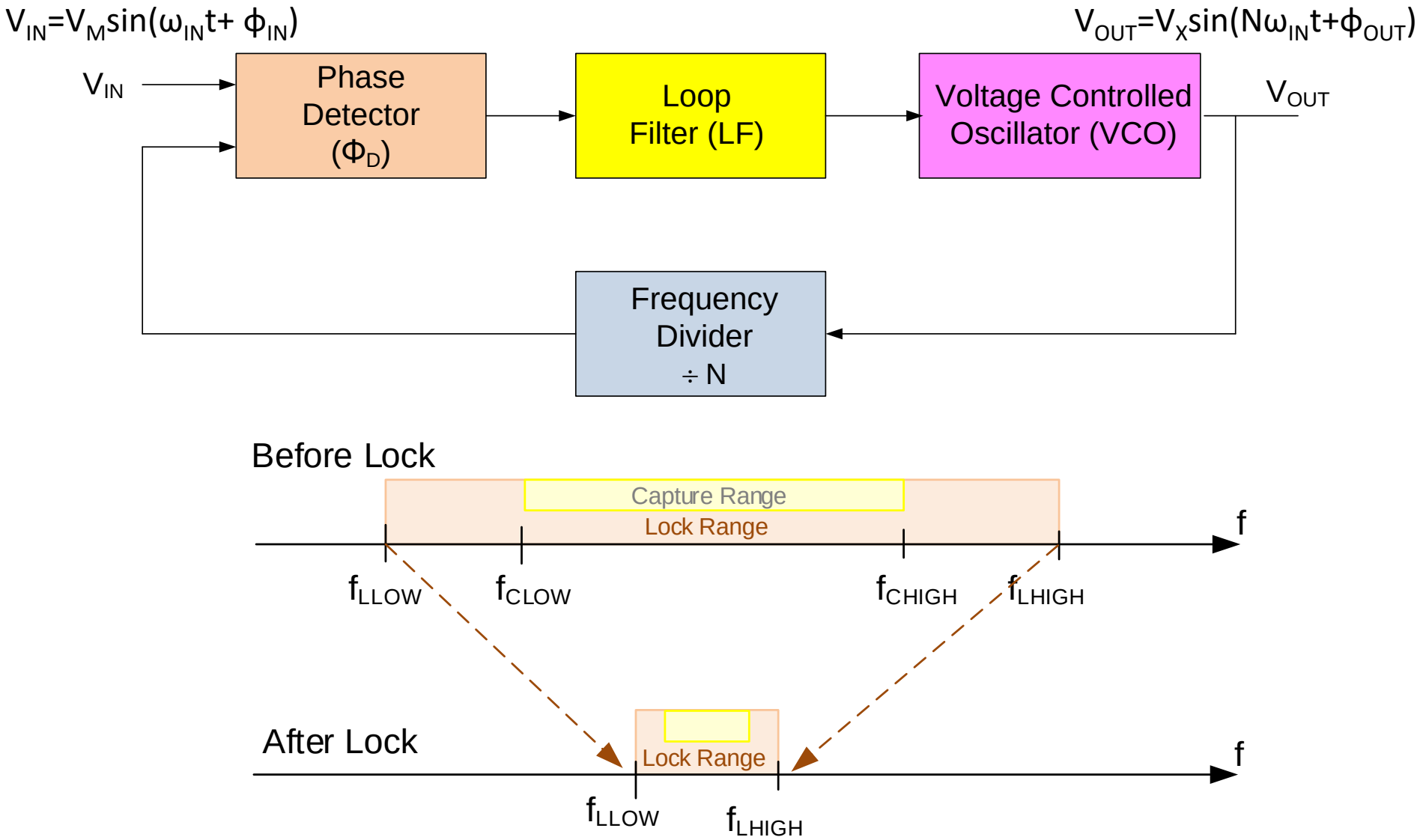


Loop filter controls capture and lock range

Jitter in VCO output strongly dependent upon lock range
(large lock range results in high jitter, low lock range in low jitter)

Loop filter is often dynamic with wide bandwidth prior to lock and narrow BW after lock

Desired Operation of PLL



Loop filter is often dynamic with wide bandwidth prior to lock and narrow BW after lock

Conceptual Operation of PLL

Consider a signal defined for $-\infty < t < \infty$ expressed as

$$V(t) = V_M \sin(\phi(t))$$

If the signal is sinusoidal with frequency ω, the argument ϕ can be expressed as

$$\phi(t)=\omega t+\theta$$

where ϕ is defined to be the phase of the signal and θ is the phase offset on the time axis from the time reference t=0

Taking the time derivative of ϕ(t) , we obtain

$$\frac{d\phi}{dt} = \omega$$

Taking the Laplace Transform, we have

$$\phi_S = \frac{\omega}{s}$$

Is the second statement “If the signal is sinusoidal ….” redundant ?

Is the second statement "If the signal is sinusoidal ...." redundant ?

Consider a signal defined for $-\infty < t < \infty$ expressed as

$$V(t) = V_M \sin(\phi)$$

If the signal is sinusoidal with frequency ω, the argument ϕ can be expressed as

$$\phi = \omega t + \theta$$

Consider any signal f(t) defined for all time (not necessarily periodic but could be)

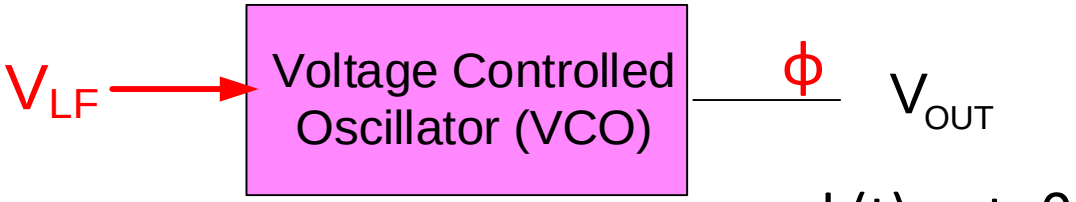
Define ϕ(t) by the expression $\phi(t) = \sin^{-1}\left(\frac{f(t)}{V_M}\right)$

It follows that $V(t) = V_M \sin(\phi(t)) = V_M \sin\left(\sin^{-1}\left(\frac{f(t)}{V_M}\right)\right) = f(t)$

Thus, the first statement gives NO information about the signal V(t)

Consider a VCO where the output is sinusoidal

Assume the output of interest is the phase ϕ



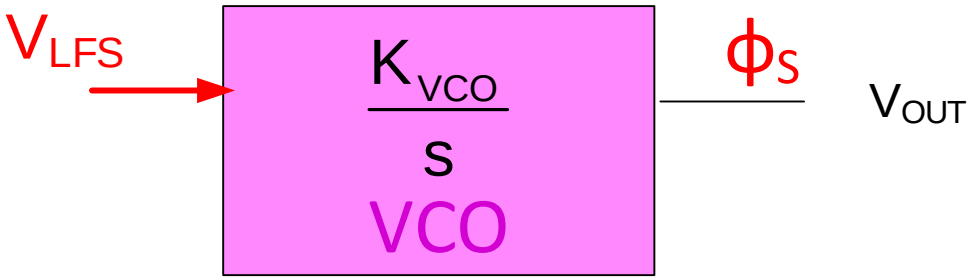
$$\phi(t)=\omega t+\theta$$

$$V_{OUT} = V_m \sin(\omega t + \theta)$$

$$\omega = V_{LF} \bullet K_{VCO}$$

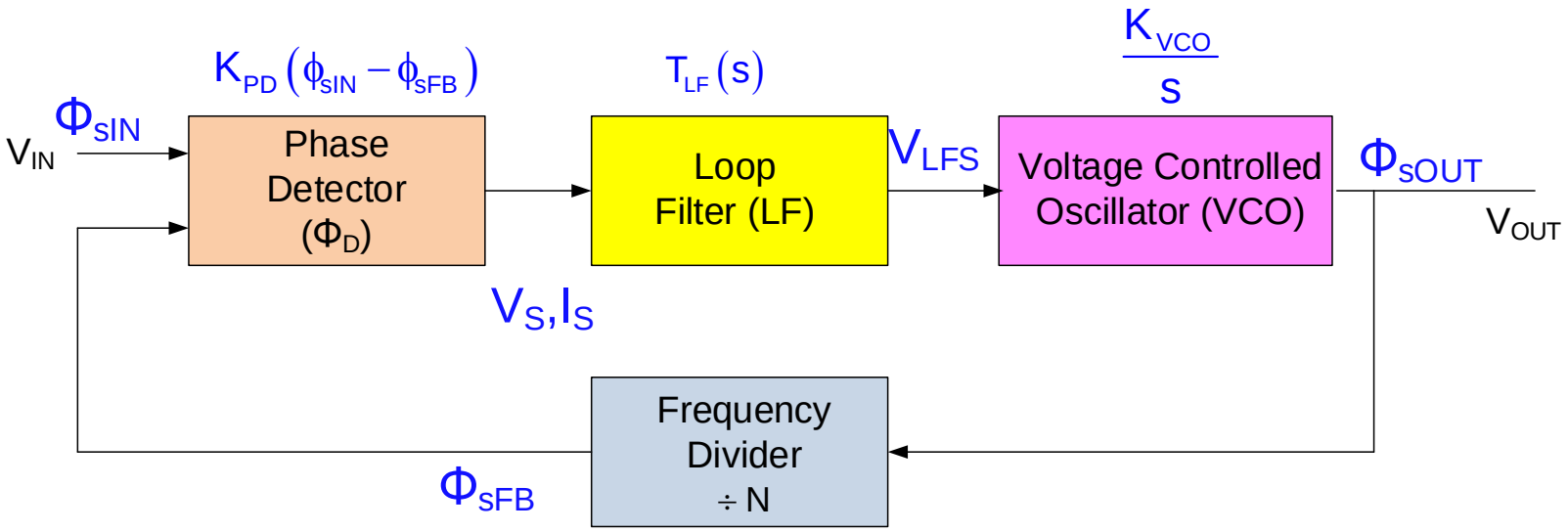
$$\frac{d\phi}{dt} = \omega = V_{LF} K_{VC0}$$

Taking Laplace Transform: $s\phi_S = V_{LFS} K_{VC0}$ ⟹ $\phi_S = V_{LFS} \frac{K_{VC0}}{s}$



Conceptual Operation of PLL

- When locked, PLL can be modeled as a linear system
- Small-signal s-domain analysis when PLL is locked



Note: Dimensions of variables in loop are not the same

$$V_S = K_{PD}\left(\phi_{sIN} - \phi_{sFB}\right)$$

$$V_{LFS} = T_{LF}(s)V_S$$

$$\phi_{sOUT} = V_{LFS}\frac{K_{VCO}}{s}$$

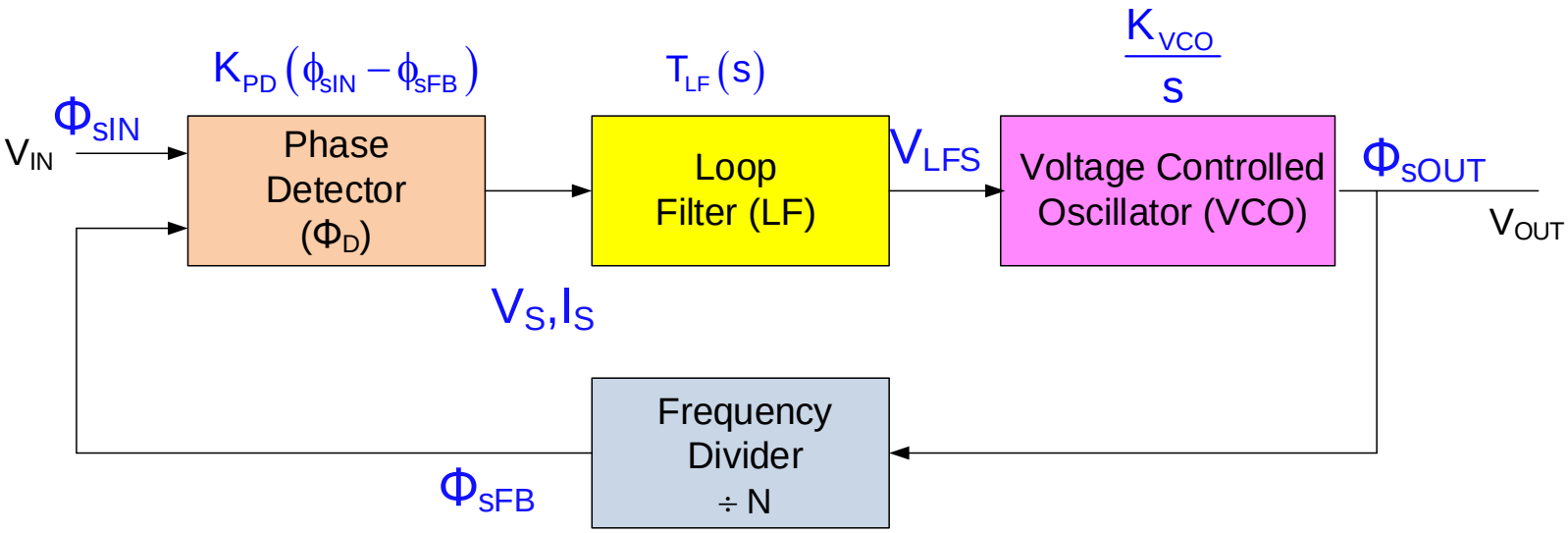
$$\phi_{sFB} = \frac{\phi_{sOUT}}{N}$$

Solving, we obtain

$$T_{PLL}(s) = \frac{\phi_{sOUT}}{\phi_{sIN}} = \frac{T_{LF}(s)K_{PD}K_{VCO}}{s + T_{LF}(s)\frac{K_{VCO}K_{PD}}{N}}$$

Often the LF is low order

Example: Assume N=1 and $T_{LF}(s) = \dfrac{1}{1+RCs}$

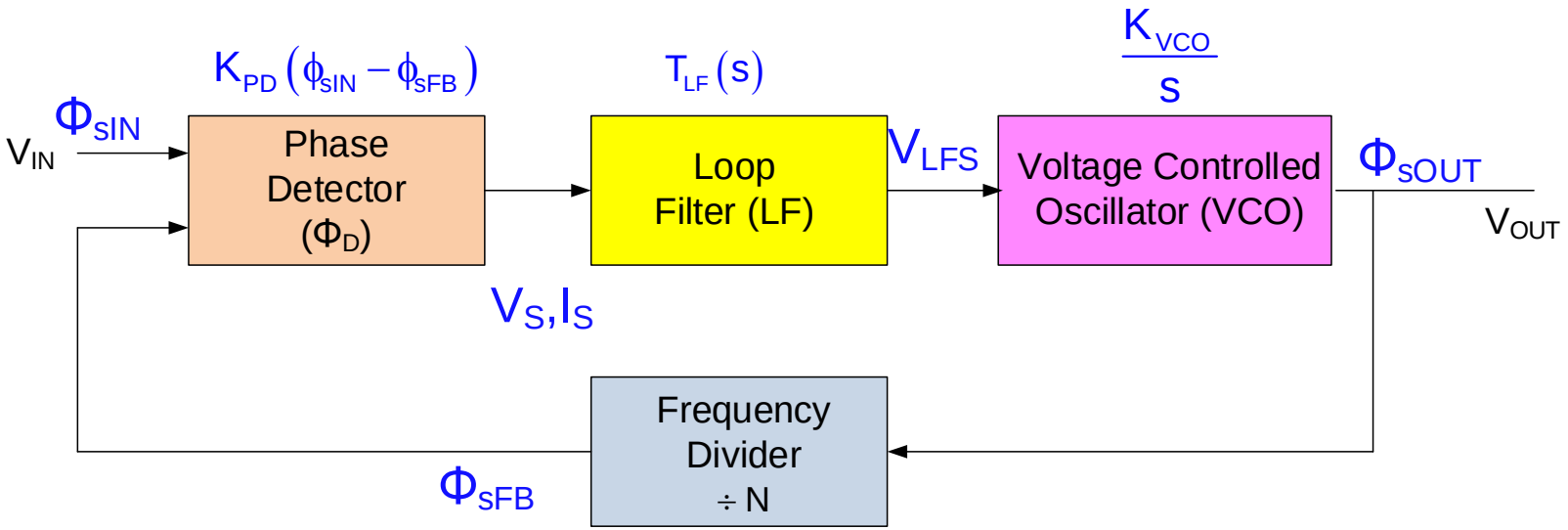


$$T_{PLL}(s) = \frac{T_{LF}(s)K_{PD}K_{VCO}}{s+T_{LF}(s)K_{VCO}K_{PD}} = \frac{K_{PD}K_{VCO}}{s(1+RCs)+K_{VCO}K_{PD}}$$

$$T_{PLL}(s) = \frac{\frac{K_{PD}K_{VCO}}{RC}}{s^2 + s\frac{1}{RC} + \frac{K_{VCO}K_{PD}}{RC}}$$

Often the LF is low order

Example: Assume N=1 and $T_{LF}(s) = \frac{1}{1+RCs}$



$$T_{PLL}(s) = \frac{\frac{K_{PD}K_{VCO}}{RC}}{s^2 + s\frac{1}{RC} + \frac{K_{VCO}K_{PD}}{RC}}$$

$$s^2 + s\frac{1}{RC} + \frac{K_{VCO}K_{PD}}{RC} = s^2 + s\frac{\omega_0}{Q} + \omega_0^2$$

$$\omega_0 = \sqrt{\frac{K_{VCO}K_{PD}}{RC}}$$

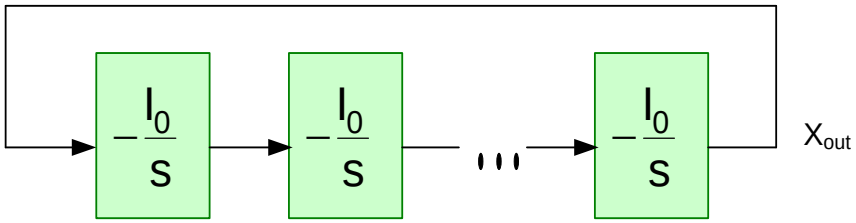
$$Q = \sqrt{RCK_{VCO}K_{PD}}$$

Voltage Controlled Oscillators

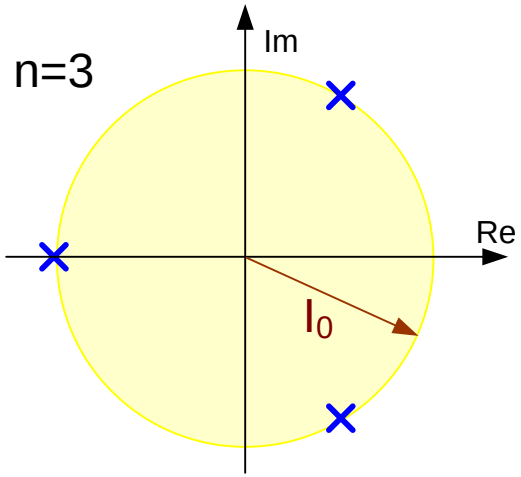
Voltage Controlled Oscillator (VCO)

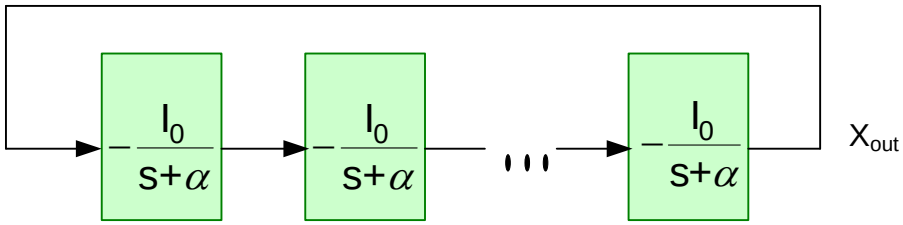
Many different VCOs can be used

Assume I_0 is determined by a controlling voltage

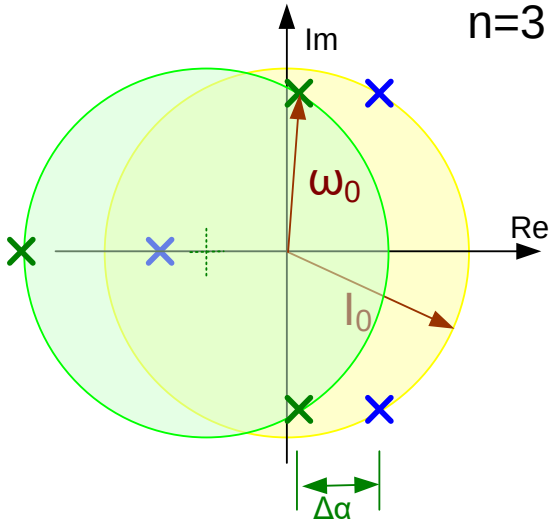


Integrator-Based VCO



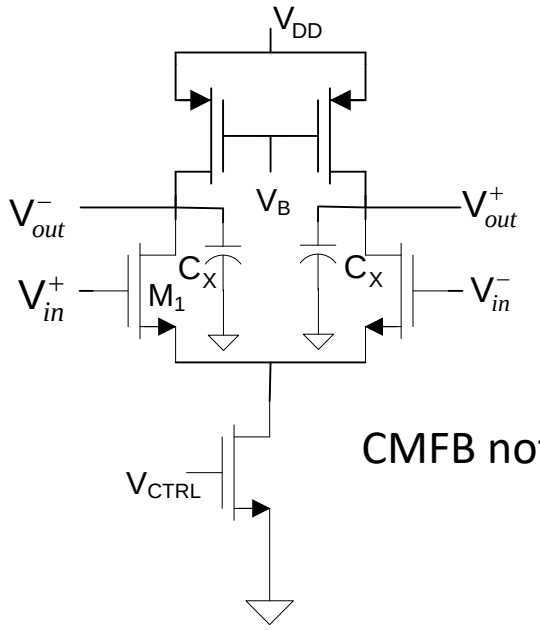


Lossy Integrator-Based VCO



Voltage Controlled Oscillators

Voltage Controlled Oscillator (VCO)

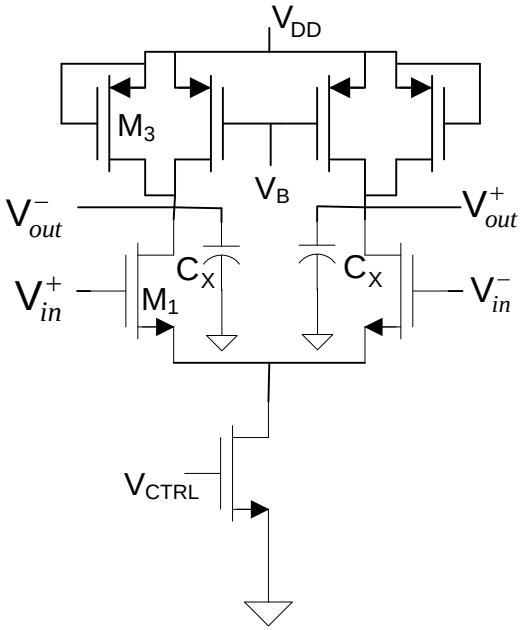


CMFB not shown

$$I_{0d}(s) = \frac{g_{m1}}{sC_X}$$

$$I_0 = \frac{g_{m1}}{C_X}$$

Integrator for: Integrator-based VCO



$$I_{0d}(s) = \frac{g_{m1}}{sC_X + g_{m3}}$$

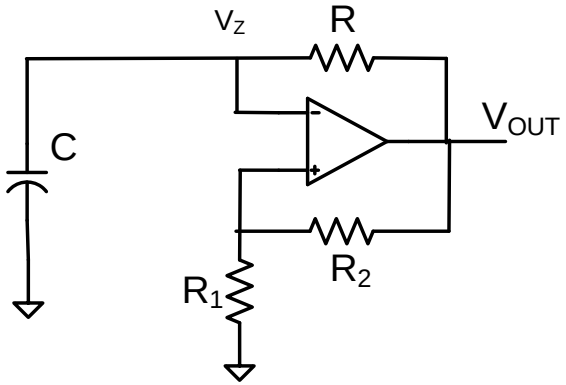
$$I_0 = \frac{g_{m1}}{C_X} \qquad \alpha = \frac{g_{m3}}{C_X}$$

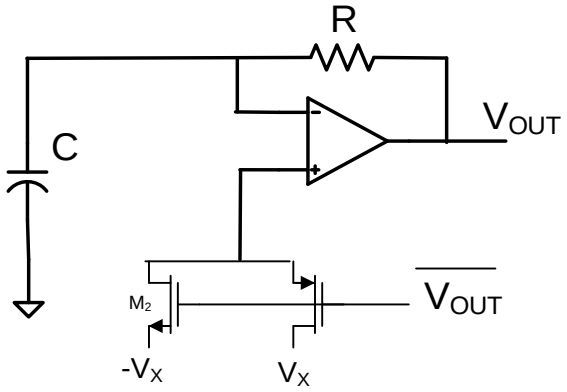
Integrator for: Lossy Integrator-based VCO

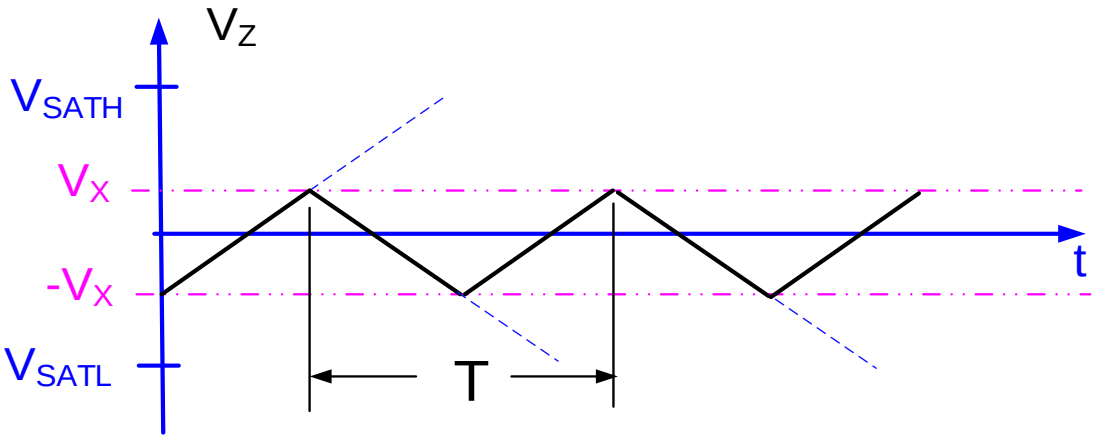
Voltage Controlled Oscillators

Voltage Controlled Oscillator (VCO)

Relaxation Oscillator Derived VCO (comparator not Op Amp)







V_Z is approximately triangle wave, V_{OUT} is square wave

Can have either triangle wave or square wave ouputs

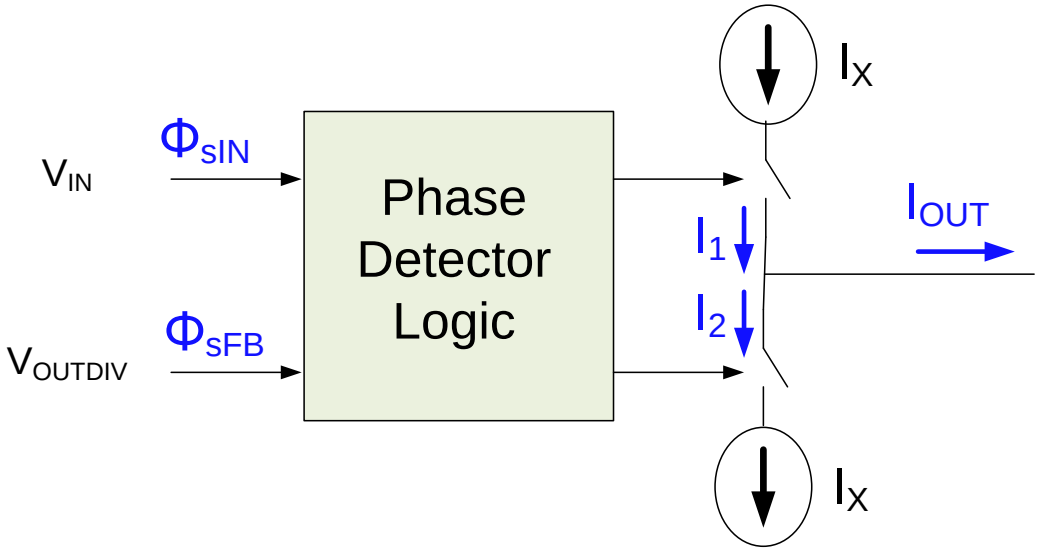
Phase Detectors

Phase Detector (Φ_D)

Many different Phase Detectors can be used

Some Popular Phase Detector Circuits

- Analog Multiplier
- Exclusive OR Gate
- Sample and Hold
- Charge Pump



Charge-pump based Phase Detector

Average I_{OUT} is the average phase

Phase Detectors

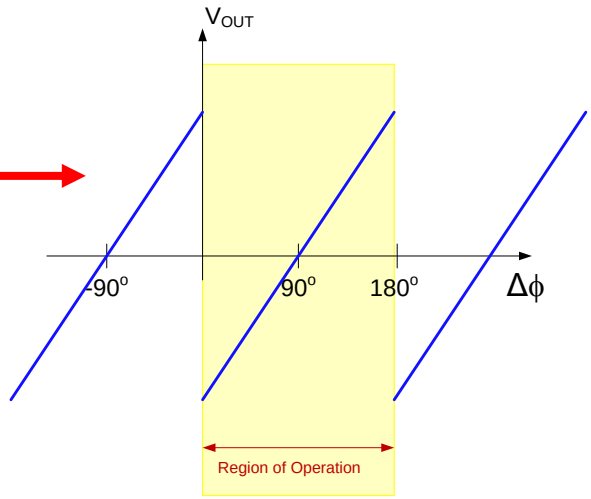
Many different Phase Detectors can be used

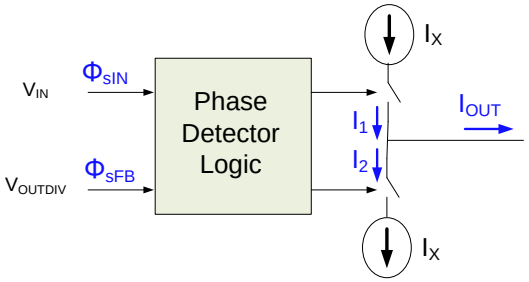
Phase Detector (Φ_D)

Desired phase difference often 0° or 90°

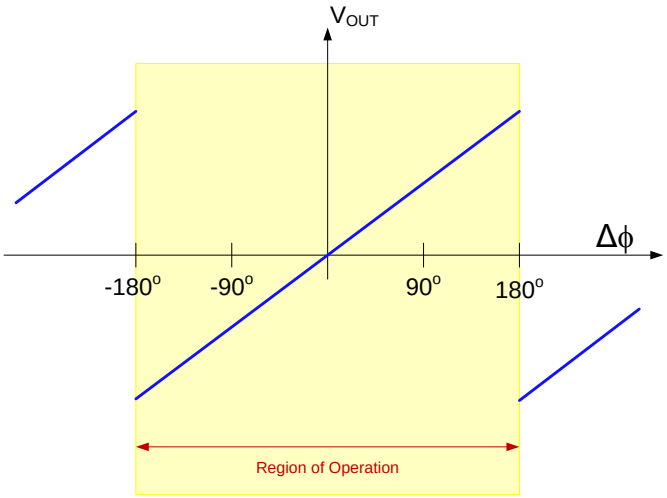
Could be simply

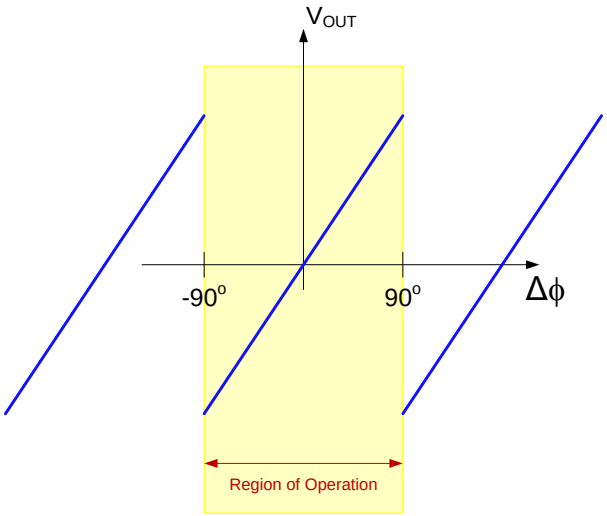
90° nominal phase diff



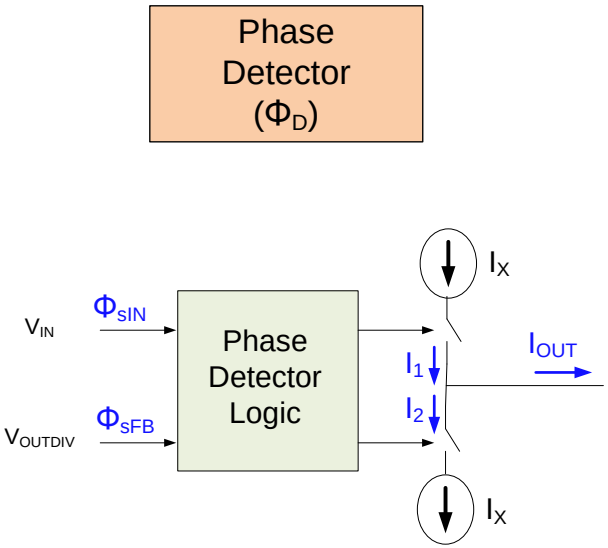


Average I_{OUT} is the average phase

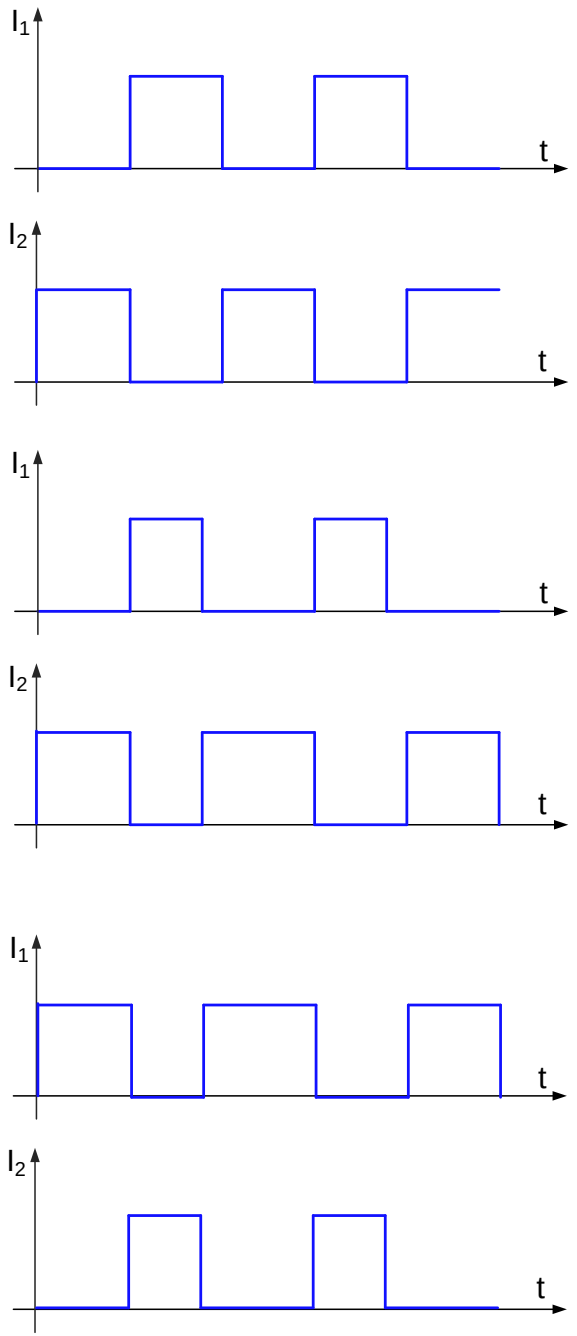




Phase Detectors

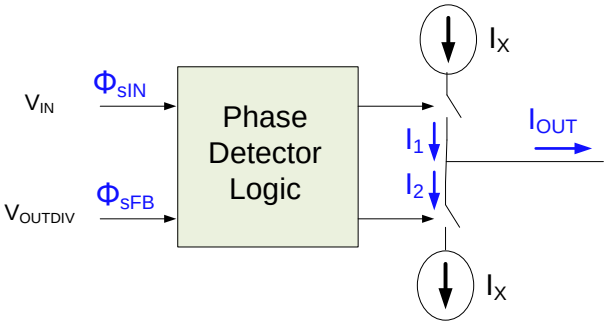


Average I_{OUT} is the average phase

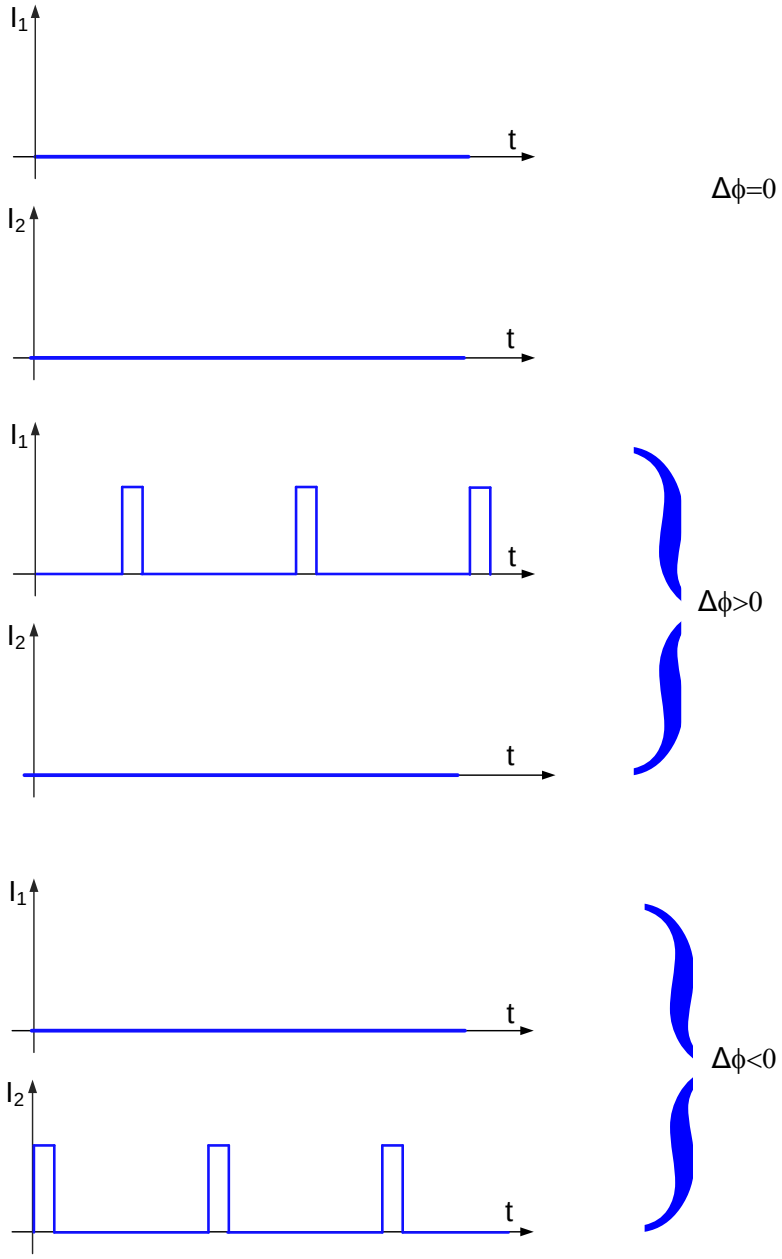


Phase Detectors

Phase Detector (Φ_D)



Average I_{OUT} is the average phase



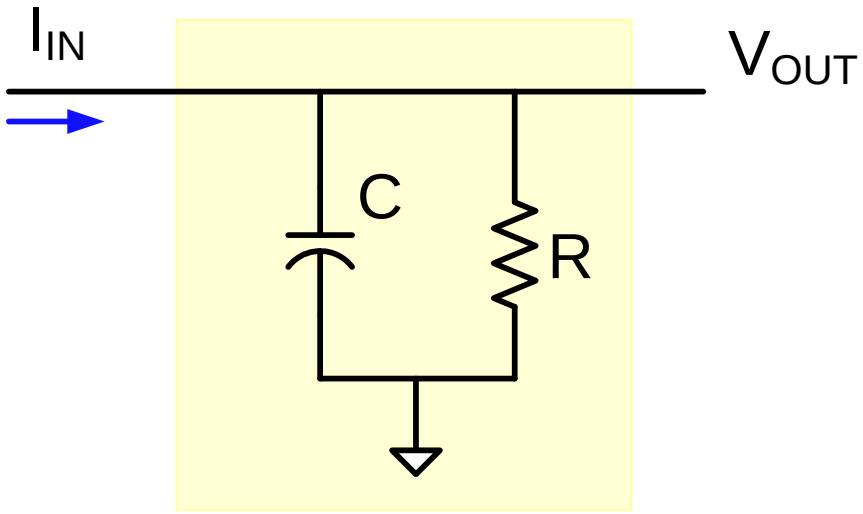
Vulnerable to dead zone problem

Loop Filters

Many different Phase Detectors can be used
Often the loop filter is first or second order
Usually the loop filter circuit is very simple

$T_{LF}(s)$

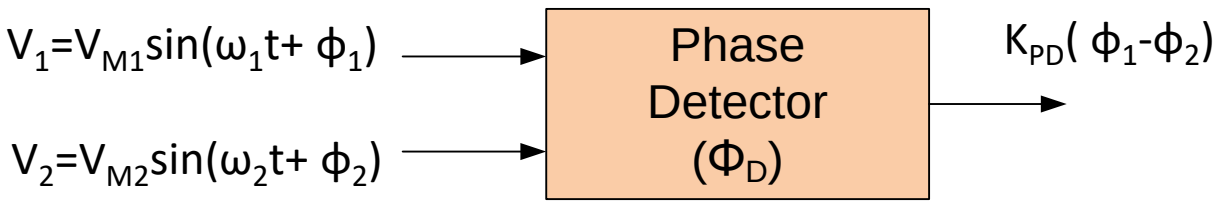
Loop Filter (LF)



$$\frac{V_{OUT}}{I_{INAVG}} = T_{LF}(s) = \frac{R}{1+RCs}$$

Basic first-order LF with average current difference as input

What is the phase of a signal?



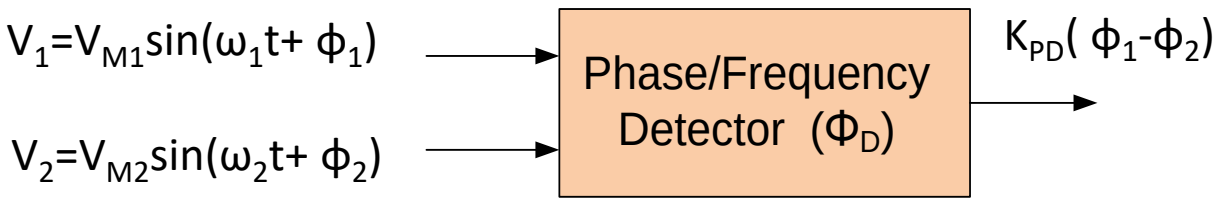
Assume $\omega_1 = \omega_2=\omega$

If V_I can be expressed as $V_1=V_{M1}\sin(\omega t+\phi_1)$ the phase is ϕ_1

But what is the phase if ω is time varying? Or what is the "phase" if this functional form does not really characterize V(t) ? Or what if $\omega_1 \neq \omega_2$?

What does a phase detector do if the two inputs are not at the same frequency?

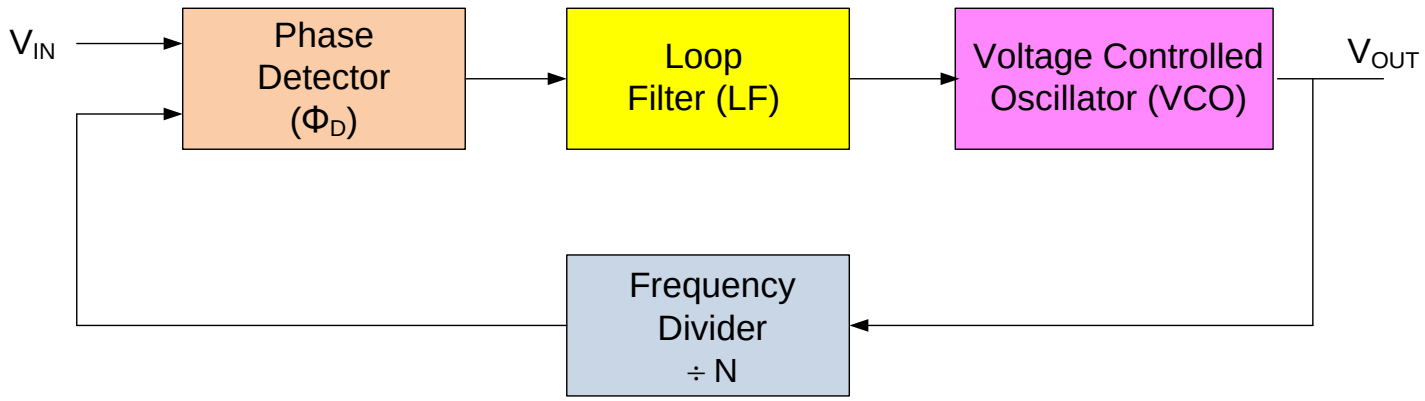
What is the phase of a signal?



Most Phase Detectors are actually Phase/Frequency Detectors

- Large output when frequency difference exists
- Also provides output when phase difference exists after frequencies are matched

Phase-Locked Loops



- Phase-Locked Loops are Widely Used
- Applications span extremely wide range of frequencies
- Invariably substantial nonlinearity complicating exact analysis
- Conceptual linear analysis gives considerable insight into operation
- Computer simulations used to mitigate nonlinarity analysis challenges

Stay Safe and Stay Healthy !

End of Lecture 43